Totten-M5. *Proposed by Ovidiu Furdui, Campia Turzii, Cluj, Romania.*

Let $a \neq 1$ be a positive real number. Determine all pairs of positive integers (x, y) such that $\log_a x - \log_a y = \log_a(x - y)$.

Solution: *(by Luis J. Blanco (student) and Angel Plaza, University of Las Palmas de Gran Canaria, 35017-Las Palmas G.C., Spain)*

$\log_a x - \log_a y = \log_a(x - y) \;\Rightarrow\; \log_a\left(\dfrac{x}{y}\right) = \log_a(x - y)$, and since $\log_a x$ is an injective function, we obtain that $\dfrac{x}{y} = x - y$.

Therefore, x is an integer multiple of $x - y$, and $x - y > 0$. Let us write $x = ny$ then $ny = y^2(n-1) \;\Rightarrow\; n = y(n-1)$, or, equivalently $y = 1 + \dfrac{1}{n-1} \in N$. That is $\dfrac{1}{n-1}$ is a positive integer, and this implies that $n = 2$. Then $y = 2$ and $x = 4$. $\square$